Forme Canoniche

Scrivere in Prima e Seconda Forma Canonica la funzione completamente specificata $f(a,b,c)$ avente ON-set $F^1=\{m_1, m_3, m_5, m_6\}$

Soluzione

Dati il numero di parametri della funzione ed il suo ON-set, possiamo scrivere la tabella di verità:

a	**b**	**c**	$f(a,b,c)$	
0	0	0	0	$M_0=a+b+c$
0	0	1	1	$m_1=\bar{a}\bar{b}c$
0	1	0	0	$M_2=a+\bar{b}+c$
0	1	1	1	$m_3=\bar{a}bc$
1	0	0	0	$M_4=\bar{a}+b+c$
1	0	1	1	$m_5=a\bar{b}c$
1	1	0	1	$m_6=ab\bar{c}$
1	1	1	0	$M_7=\bar{a}+\bar{b}+\bar{c}$

Prima Forma Canonica (PFC):

$$f(a,b,c)=m_1+m_3+m_5+m_6=\bar{a}\bar{b}c+\bar{a}bc+a\bar{b}c+ab\bar{c}$$

Seconda Forma Canonica (SFC):

$$f(a,b,c)=M_0\cdot M_2\cdot M_4\cdot M_7=(a+b+c)\cdot(a+\bar{b}+c)\cdot(\bar{a}+b+c)\cdot(\bar{a}+\bar{b}+\bar{c})$$

Mappe di Karnaugh

1. $f(a,b,c)=\sum m(2,3,4,5)$ => $f(a,b,c)=\bar{a}b+a\bar{b}$

a \ bc	00	01	11	10
0	0	0	1	1
1	1	1	0	0

2. $f(a,b,c)=\sum m(3,4,6,7)$ => $f(a,b,c)=a\bar{c}+bc$

a \ bc	00	01	11	10
0	0	0	1	0
1	1	0	1	1

3. $f(a,b,c)=\sum m(0,2,4,5,6)$ => $f(a,b,c)=\bar{c}+a\bar{b}$

a \ bc	00	01	11	10
0	1	0	0	1
1	1	1	0	1

4. $f(a,b,c)=\sum m(1,3,4,5,6)$ => $f(a,b,c)=a\bar{c}+\bar{a}c+\bar{b}c$
oppure $f(a,b,c)=a\bar{c}+\bar{a}c+a\bar{b}$

a \ bc	00	01	11	10
0	0	1	1	0
1	1	1	0	1

a \ bc	00	01	11	10
0	0	1	1	0
1	1	1	0	1

5. $f(a,b,c)=\bar{a}c+\bar{a}b+a\bar{b}c+bc \quad => \quad f(a,b,c)=c+\bar{a}b$

a \ bc	00	01	11	10
0	0	1	1	1
1	0	1	1	0

6. $f(a,b,c,d)=\sum m(0,1,2,4,5,6,8,9,12,13,14) \quad => \quad f(a,b,c,d)=\bar{c}+\bar{a}\bar{d}+b\bar{d}$

ab \ cd	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

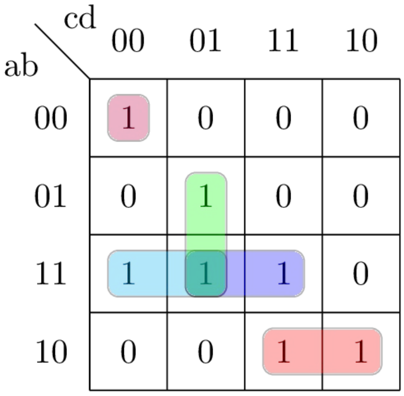
7. $f(a,b,c,d)=\bar{a}\bar{b}\bar{c}+\bar{b}c\bar{d}+a\bar{b}\bar{c}+\bar{a}bc\bar{d} \quad => \quad f(a,b,c,d)=\bar{b}\bar{d}+\bar{b}\bar{c}+\bar{a}c\bar{d}$

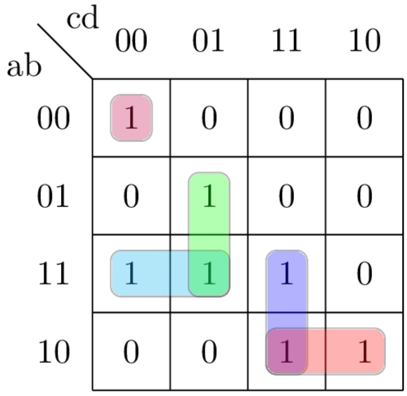
ab \ cd	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	0	0	0	0
10	1	1	0	1

8. $f(a,b,c,d)=\sum m(0,5,10,11,12,13,15)$

$\Rightarrow \quad f(a,b,c,d)=\bar{a}\bar{b}\bar{c}\bar{d}+ab\bar{c}+b\bar{c}d+a\bar{b}c+abd$

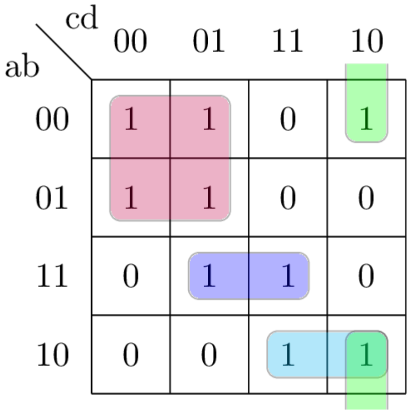
oppure $\quad f(a,b,c,d)=\bar{a}\bar{b}\bar{c}\bar{d}+ab\bar{c}+b\bar{c}d+a\bar{b}c+acd$



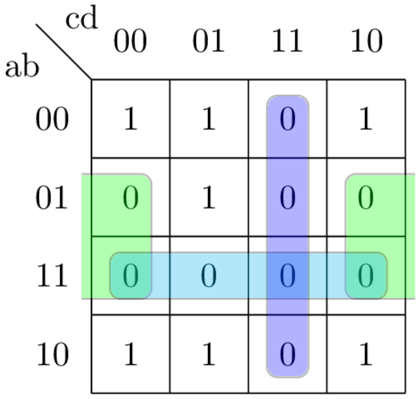


9. $f(a,b,c,d)=\sum m(0,1,2,4,5,10,11,13,15)$

una possibile forma minima è $\quad f(a,b,c,d)=\bar{a}\bar{c}+abd+a\bar{b}c+\bar{b}c\bar{d}$



10. $f(a,b,c,d)=\prod M(3,4,6,7,11,12,13,14,15)$ procedendo in maniera duale rispetto ai casi visti fino a questo momento si ha: $\quad f(a,b,c,d)=(\bar{b}+d)(\bar{a}+\bar{b})(\bar{c}+\bar{d})$



11. $f(a,b,c,d)=\sum m(1,3,7,11,15)+\sum d(0,2,5)$

=> $f(a,b,c,d)=\bar{a}\bar{b}+cd$

oppure $f(a,b,c,d)=\bar{a}d+cd$

ab \ cd	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

ab \ cd	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

12. Minimizzare la funzione rappresentata dalla mappa:

bc \ de	00	01	11	10
00	1	X	1	X
01	0	0	1	1
11	0	X	0	0
10	0	1	1	0

a=0

bc \ de	00	01	11	10
00	X	1	1	1
01	0	X	0	X
11	0	0	1	1
10	X	1	X	0

a=1

=> $f(a,b,c,d,e)=\bar{b}\bar{c}+\bar{c}e+\bar{a}\bar{b}d+abcd$

bc \ de	00	01	11	10
00	1	X	1	X
01	0	0	1	1
11	0	X	0	0
10	0	1	1	0

a=0

bc \ de	00	01	11	10
00	X	1	1	1
01	0	X	0	X
11	0	0	1	1
10	X	1	X	0

a=1

13. Si considerino i numeri composti da 5 bit (a, b, c, d, e) con a il bit più significativo.
Si determini:
   1. la funzione booleana che vale 1 solo quando il numero ha valore pari ad 1 o ad un numero primo;
   2. l'espressione minima, nelle forme SP e PS, della funzione booleana determinata al punto precedente.

Soluzione

1. $f(a, b, c, d, e) = \sum m(1,2,3,5,7,11,13,17,19,23,29,31)$

2. Espressione minima nella forma SP

$$f(a, b, c, d, e) = \sum m(1,2,3,5,7,11,13,17,19,23,29,31)$$
$$\Rightarrow \quad f(a, b, c, d, e) = \bar{b}\bar{c}e + \bar{b}de + \bar{a}\bar{b}\bar{c}d + \bar{a}c\bar{d}e + \bar{a}\bar{c}de + abce$$

a=0

bc \ de	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	1	0	0
10	0	0	1	0

a=1

bc \ de	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	1	1	0
10	0	0	0	0

Espressione minima nella forma PS

$$f(a, b, c, d, e) = \prod M(0,4,6,8,9,10,12,14,15,16,18,20,21,22,24,25,26,27,28,30) \Rightarrow$$

$$f(a, b, c, d, e) = (d+e)(\bar{c}+e)(\bar{b}+e)(\bar{b}+c+d)(a+\bar{b}+\bar{c}+\bar{d})(\bar{a}+e)(\bar{a}+b+\bar{c}+d)(\bar{a}+\bar{b}+c)$$

a=0

bc \ de	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	1	0	0
10	0	0	1	0

a=1

bc \ de	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	1	1	0
10	0	0	0	0