

## Forme Canoniche

1. Scrivere in Prima e Seconda Forma Canonica la funzione completamente specificata  $f(a, b, c)$  avente ON-set  $F^1 = \{m_1, m_3, m_5, m_6\}$

*Soluzione*

Dati il numero di parametri della funzione ed il suo ON-set, possiamo scrivere la tabella di verità:

| a | b | c | $f(a, b, c)$ |                                     |
|---|---|---|--------------|-------------------------------------|
| 0 | 0 | 0 | 0            | $M_0 = a + b + c$                   |
| 0 | 0 | 1 | 1            | $m_1 = \bar{a}\bar{b}c$             |
| 0 | 1 | 0 | 0            | $M_2 = a + \bar{b} + c$             |
| 0 | 1 | 1 | 1            | $m_3 = \bar{a}bc$                   |
| 1 | 0 | 0 | 0            | $M_4 = \bar{a} + b + c$             |
| 1 | 0 | 1 | 1            | $m_5 = a\bar{b}c$                   |
| 1 | 1 | 0 | 1            | $m_6 = ab\bar{c}$                   |
| 1 | 1 | 1 | 0            | $M_7 = \bar{a} + \bar{b} + \bar{c}$ |

**Prima Forma Canonica (PFC):**

$$f(a, b, c) = m_1 + m_3 + m_5 + m_6 = \bar{a}\bar{b}c + \bar{a}bc + a\bar{b}c + ab\bar{c}$$

**Seconda Forma Canonica (SFC):**

$$f(a, b, c) = M_0 \cdot M_2 \cdot M_4 \cdot M_7 = (a + b + c) \cdot (a + \bar{b} + c) \cdot (\bar{a} + b + c) \cdot (\bar{a} + \bar{b} + \bar{c})$$

2. Scrivere in Prima e Seconda Forma Canonica la funzione completamente specificata  $f(a, b, c)$  avente ON-set  $F^1 = \{m_1, m_4, m_5, m_7\}$

*Soluzione*

Dati il numero di parametri della funzione ed il suo ON-set, possiamo scrivere la tabella di verità:

| <b>a</b> | <b>b</b> | <b>c</b> | $f(a, b, c)$ |                               |
|----------|----------|----------|--------------|-------------------------------|
| 0        | 0        | 0        | 0            | $M_0 = a + b + c$             |
| 0        | 0        | 1        | 1            | $m_1 = \bar{a}\bar{b}c$       |
| 0        | 1        | 0        | 0            | $M_2 = a + \bar{b} + c$       |
| 0        | 1        | 1        | 0            | $M_3 = a + \bar{b} + \bar{c}$ |
| 1        | 0        | 0        | 1            | $m_4 = a\bar{b}\bar{c}$       |
| 1        | 0        | 1        | 1            | $m_5 = a\bar{b}c$             |
| 1        | 1        | 0        | 0            | $M_6 = \bar{a} + \bar{b} + c$ |
| 1        | 1        | 1        | 1            | $m_7 = abc$                   |

**Prima Forma Canonica (PFC):**

$$f(a, b, c) = m_1 + m_4 + m_5 + m_7 = \bar{a}\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + abc$$

**Seconda Forma Canonica (SFC):**

$$f(a, b, c) = M_0 \cdot M_2 \cdot M_3 \cdot M_6 = (a + b + c) \cdot (a + \bar{b} + c) \cdot (a + \bar{b} + \bar{c}) \cdot (\bar{a} + \bar{b} + c)$$

## Mappe di Karnaugh

1.  $f(a, b, c) = \sum m(2, 3, 4, 5) \Rightarrow f(a, b, c) = \bar{a}b + a\bar{b}$

|   |   |    |    |    |    |
|---|---|----|----|----|----|
|   |   | bc |    |    |    |
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 0  | 0  | 1  | 1  |
|   | 1 | 1  | 1  | 0  | 0  |

2.  $f(a, b, c) = \sum m(0, 2, 3, 4) \Rightarrow f(a, b, c) = \bar{a}b + \bar{b}\bar{c}$

|   |   |    |    |    |    |
|---|---|----|----|----|----|
|   |   | bc |    |    |    |
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 0  | 1  | 1  |
|   | 1 | 1  | 0  | 0  | 0  |

3.  $f(a, b, c) = \sum m(3, 4, 6, 7) \Rightarrow f(a, b, c) = a\bar{c} + bc$

|   |   |    |    |    |    |
|---|---|----|----|----|----|
|   |   | bc |    |    |    |
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 0  | 0  | 1  | 0  |
|   | 1 | 1  | 0  | 1  | 1  |

4.  $f(a, b, c) = \sum m(0, 2, 3, 4, 6) \Rightarrow f(a, b, c) = \bar{c} + \bar{a}b$

|   |   |    |    |    |    |
|---|---|----|----|----|----|
|   |   | bc |    |    |    |
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 0  | 1  | 1  |
|   | 1 | 1  | 0  | 0  | 1  |

5.  $f(a,b,c) = \sum m(0,2,4,5,6) \Rightarrow f(a,b,c) = \bar{c} + a\bar{b}$

| a \ bc | 00 | 01 | 11 | 10 |
|--------|----|----|----|----|
|        | 0  | 1  | 1  | 0  |
| 0      | 1  | 0  | 0  | 1  |
| 1      | 1  | 1  | 0  | 1  |

6.  $f(a,b,c) = \sum m(1,3,4,5,6) \Rightarrow$  oppure  $f(a,b,c) = a\bar{c} + \bar{a}c + \bar{b}c$   
 $f(a,b,c) = a\bar{c} + \bar{a}c + a\bar{b}$

| a \ bc | 00 | 01 | 11 | 10 |
|--------|----|----|----|----|
|        | 0  | 1  | 1  | 0  |
| 0      | 0  | 1  | 1  | 0  |
| 1      | 1  | 1  | 0  | 1  |

| a \ bc | 00 | 01 | 11 | 10 |
|--------|----|----|----|----|
|        | 0  | 1  | 1  | 0  |
| 0      | 0  | 1  | 1  | 0  |
| 1      | 1  | 1  | 0  | 1  |

7.  $f(a,b,c) = \bar{a}c + \bar{a}b + a\bar{b}c + bc \Rightarrow f(a,b,c) = c + \bar{a}b$

| a \ bc | 00 | 01 | 11 | 10 |
|--------|----|----|----|----|
|        | 0  | 1  | 1  | 0  |
| 0      | 0  | 1  | 1  | 1  |
| 1      | 0  | 1  | 1  | 0  |

8.  $f(a,b,c,d) = \sum m(0,1,2,4,5,6,8,9,12,13,14) \Rightarrow f(a,b,c,d) = \bar{c} + \bar{a}\bar{d} + b\bar{d}$

| ab \ cd | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
|         | 0  | 1  | 1  | 0  |
| 00      | 1  | 1  | 0  | 1  |
| 01      | 1  | 1  | 0  | 1  |
| 11      | 1  | 1  | 0  | 1  |
| 10      | 1  | 1  | 0  | 0  |

9.  $f(a,b,c,d) = \sum m(0,1,3,8,9,10,11,12,13,14,15) \Rightarrow f(a,b,c,d) = a + \bar{b}\bar{c} + \bar{b}d$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | 1  | 0  |
| 01      | 0  | 0  | 0  | 0  |
| 11      | 1  | 1  | 1  | 1  |
| 10      | 1  | 1  | 1  | 1  |

10.  $f(a,b,c,d) = \bar{a}\bar{b}\bar{c} + \bar{b}c\bar{d} + a\bar{b}\bar{c} + \bar{a}bcd \Rightarrow f(a,b,c,d) = \bar{b}\bar{d} + \bar{b}\bar{c} + \bar{a}c\bar{d}$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | 0  | 1  |
| 01      | 0  | 0  | 0  | 1  |
| 11      | 0  | 0  | 0  | 0  |
| 10      | 1  | 1  | 0  | 1  |

11.  $f(a,b,c,d) = \sum m(0,5,10,11,12,13,15)$   
 $\Rightarrow f(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + ab\bar{c} + b\bar{c}d + a\bar{b}c + abd$   
oppure  $f(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + ab\bar{c} + b\bar{c}d + a\bar{b}c + acd$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 0  | 0  | 0  |
| 01      | 0  | 1  | 0  | 0  |
| 11      | 1  | 1  | 1  | 0  |
| 10      | 0  | 0  | 1  | 1  |

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 0  | 0  | 0  |
| 01      | 0  | 1  | 0  | 0  |
| 11      | 1  | 1  | 1  | 0  |
| 10      | 0  | 0  | 1  | 1  |

12.  $f(a, b, c, d) = \sum m(0, 2, 5, 8, 10, 11, 12, 13, 15)$

$\Rightarrow f(a, b, c, d) = \bar{b}\bar{d} + ab\bar{c} + b\bar{c}d + a\bar{b}c + abd$

oppure  $f(a, b, c, d) = \bar{b}\bar{d} + ab\bar{c} + b\bar{c}d + a\bar{b}c + acd$

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | cd |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | 1  | 0  | 0  | 1  |
|    | 01 | 0  | 1  | 0  | 0  |
|    | 11 | 1  | 1  | 1  | 0  |
|    | 10 | 1  | 0  | 1  | 1  |

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | cd |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | 1  | 0  | 0  | 1  |
|    | 01 | 0  | 1  | 0  | 0  |
|    | 11 | 1  | 1  | 1  | 0  |
|    | 10 | 1  | 0  | 1  | 1  |

13.  $f(a, b, c, d) = \sum m(0, 1, 2, 4, 5, 10, 11, 13, 15)$

una possibile forma minima è  $f(a, b, c, d) = \bar{a}\bar{c} + abd + a\bar{b}c + \bar{b}c\bar{d}$

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | cd |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | 1  | 1  | 0  | 1  |
|    | 01 | 1  | 1  | 0  | 0  |
|    | 11 | 0  | 1  | 1  | 0  |
|    | 10 | 0  | 0  | 1  | 1  |

14.  $f(a, b, c, d) = \prod M(3, 7, 11, 15)$

Procedendo in maniera duale rispetto ai casi visti fino a questo momento (riguardanti la Somma di Prodotti) si ha:  $f(a, b, c, d) = (\bar{c} + \bar{d})$

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | cd |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | 1  | 1  | 0  | 1  |
|    | 01 | 1  | 1  | 0  | 1  |
|    | 11 | 1  | 1  | 0  | 1  |
|    | 10 | 1  | 1  | 0  | 1  |

15.  $f(a, b, c, d) = \prod M(3, 4, 6, 7, 11, 12, 13, 14, 15)$

Procedendo come nel caso precedente, in maniera duale rispetto alla Somma di Prodotti, si ha:  $f(a, b, c, d) = (\bar{b} + d)(\bar{a} + \bar{b})(\bar{c} + \bar{d})$

|    |    | cd |    |    |    |
|----|----|----|----|----|----|
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | 1  | 1  | 0  | 1  |
|    | 01 | 0  | 1  | 0  | 0  |
|    | 11 | 0  | 0  | 0  | 0  |
|    | 10 | 1  | 1  | 0  | 1  |

16.  $f(a, b, c, d) = \sum m(1, 3, 7, 11, 15) + \sum d(0, 2, 5)$

$\Rightarrow f(a, b, c, d) = \bar{a}\bar{b} + cd$

oppure  $f(a, b, c, d) = \bar{a}d + cd$

|    |    | cd |    |    |    |
|----|----|----|----|----|----|
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | X  | 1  | 1  | X  |
|    | 01 | 0  | X  | 1  | 0  |
|    | 11 | 0  | 0  | 1  | 0  |
|    | 10 | 0  | 0  | 1  | 0  |

|    |    | cd |    |    |    |
|----|----|----|----|----|----|
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | X  | 1  | 1  | X  |
|    | 01 | 0  | X  | 1  | 0  |
|    | 11 | 0  | 0  | 1  | 0  |
|    | 10 | 0  | 0  | 1  | 0  |

17.  $f(a, b, c, d) = \sum m(0, 1, 2, 3, 7, 15) + \sum d(11, 13)$

$\Rightarrow f(a, b, c, d) = \bar{a}\bar{b} + cd$

|    |    | cd |    |    |    |
|----|----|----|----|----|----|
|    |    | 00 | 01 | 11 | 10 |
| ab | 00 | 1  | 1  | 1  | 1  |
|    | 01 | 0  | 0  | 1  | 0  |
|    | 11 | 0  | X  | 1  | 0  |
|    | 10 | 0  | 0  | X  | 0  |

18.  $f(a, b, c, d) = \sum m(0,1,2,3,7,15) + \sum d(8,10,11,13)$   
 $\Rightarrow f(a, b, c, d) = \bar{a}\bar{b} + cd$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | 1  | 1  |
| 01      | 0  | 0  | 1  | 0  |
| 11      | 0  | X  | 1  | 0  |
| 10      | X  | 0  | X  | X  |

19.  $f(a, b, c, d) = \sum m(0,1,2,3,7,15) + \sum d(8,9,10,11,13)$   
 $\Rightarrow f(a, b, c, d) = \bar{a} + cd$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | 1  | 1  |
| 01      | 0  | 0  | 1  | 0  |
| 11      | 0  | X  | 1  | 0  |
| 10      | X  | X  | X  | X  |

20.  $f(a, b, c, d) = \sum m(4,12,13,14,15) + \sum d(0,2,5,8)$   
 $\Rightarrow f(a, b, c, d) = \bar{c}\bar{d} + ab$   
 oppure  $f(a, b, c, d) = b\bar{c} + ab$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | X  | 0  | 0  | X  |
| 01      | 1  | X  | 0  | 0  |
| 11      | 1  | 1  | 1  | 1  |
| 10      | X  | 0  | 0  | 0  |

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | X  | 0  | 0  | X  |
| 01      | 1  | X  | 0  | 0  |
| 11      | 1  | 1  | 1  | 1  |
| 10      | X  | 0  | 0  | 0  |



21. Minimizzare la funzione rappresentata dalla mappa:

| bc \ de | 00 01 11 10 |    |    |    |
|---------|-------------|----|----|----|
|         | 00          | 01 | 11 | 10 |
| 00      | 1           | X  | 1  | X  |
| 01      | 0           | 0  | 1  | 1  |
| 11      | 0           | X  | 0  | 0  |
| 10      | 0           | 1  | 1  | 0  |

a=0

| bc \ de | 00 01 11 10 |    |    |    |
|---------|-------------|----|----|----|
|         | 00          | 01 | 11 | 10 |
| 00      | X           | 1  | 1  | 1  |
| 01      | 0           | X  | 0  | X  |
| 11      | 0           | 0  | 1  | 1  |
| 10      | X           | 1  | X  | 0  |

a=1

$$\Rightarrow f(a, b, c, d, e) = \bar{b}\bar{c} + \bar{c}e + \bar{a}\bar{b}d + abcd$$

| bc \ de | 00 01 11 10 |    |    |    |
|---------|-------------|----|----|----|
|         | 00          | 01 | 11 | 10 |
| 00      | 1           | X  | 1  | X  |
| 01      | 0           | 0  | 1  | 1  |
| 11      | 0           | X  | 0  | 0  |
| 10      | 0           | 1  | 1  | 0  |

a=0

| bc \ de | 00 01 11 10 |    |    |    |
|---------|-------------|----|----|----|
|         | 00          | 01 | 11 | 10 |
| 00      | X           | 1  | 1  | 1  |
| 01      | 0           | X  | 0  | X  |
| 11      | 0           | 0  | 1  | 1  |
| 10      | X           | 1  | X  | 0  |

a=1

22. Si consideri un sistema composto da tre interruttori collegati ad una sola lampadina. Si ricavi la forma minima della funzione logica per cui la lampadina è accesa solo se almeno due interruttori sono chiusi.

*Soluzione*

| a | b | c | $f(a, b, c)$ |                         |
|---|---|---|--------------|-------------------------|
| 0 | 0 | 0 | 0            | $M_0 = a + b + c$       |
| 0 | 0 | 1 | 0            | $M_1 = a + b + \bar{c}$ |
| 0 | 1 | 0 | 0            | $M_2 = a + \bar{b} + c$ |
| 0 | 1 | 1 | 1            | $m_3 = \bar{a}bc$       |
| 1 | 0 | 0 | 0            | $M_4 = \bar{a} + b + c$ |
| 1 | 0 | 1 | 1            | $m_5 = a\bar{b}c$       |
| 1 | 1 | 0 | 1            | $m_6 = ab\bar{c}$       |
| 1 | 1 | 1 | 1            | $m_7 = abc$             |

| a \ bc | 00 01 11 10 |    |    |    |
|--------|-------------|----|----|----|
|        | 00          | 01 | 11 | 10 |
| 0      | 0           | 0  | 1  | 0  |
| 1      | 0           | 1  | 1  | 1  |

$$f(a, b, c) = bc + ab + a\bar{c}$$

23. Si considerino i numeri composti da 5 bit ( $a, b, c, d, e$ ) con  $a$  il bit più significativo.  
Si determini:

1. la funzione booleana che vale 1 solo quando il numero ha valore pari ad 1 o ad un numero primo;
2. l'espressione minima, nelle forme SP e PS, della funzione booleana determinata al punto precedente.

*Soluzione*

1.  $f(a, b, c, d, e) = \sum m(1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31)$

2. Espressione minima nella forma SP

$$f(a, b, c, d, e) = \sum m(1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31)$$

$$\Rightarrow f(a, b, c, d, e) = \bar{b}\bar{c}e + \bar{b}de + \bar{a}\bar{b}\bar{c}d + \bar{a}c\bar{d}e + \bar{a}\bar{c}de + abce$$

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | de |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| bc | 00 | 0  | 1  | 1  | 1  |
|    | 01 | 0  | 1  | 1  | 0  |
|    | 11 | 0  | 1  | 0  | 0  |
|    | 10 | 0  | 0  | 1  | 0  |

a=0

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | de |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| bc | 00 | 0  | 1  | 1  | 0  |
|    | 01 | 0  | 0  | 1  | 0  |
|    | 11 | 0  | 1  | 1  | 0  |
|    | 10 | 0  | 0  | 0  | 0  |

a=1

Espressione minima nella forma PS

$$f(a, b, c, d, e) = \prod M(0, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30) \Rightarrow$$

$$f(a, b, c, d, e) = (d + e)(\bar{c} + e)(\bar{b} + e)(\bar{b} + c + d)(a + \bar{b} + \bar{c} + \bar{d})(\bar{a} + e)(\bar{a} + b + \bar{c} + d)(\bar{a} + \bar{b} + c)$$

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | de |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| bc | 00 | 0  | 1  | 1  | 1  |
|    | 01 | 0  | 1  | 1  | 0  |
|    | 11 | 0  | 1  | 0  | 0  |
|    | 10 | 0  | 0  | 1  | 0  |

a=0

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    |    | de |    |    |    |
|    |    | 00 | 01 | 11 | 10 |
| bc | 00 | 0  | 1  | 1  | 0  |
|    | 01 | 0  | 0  | 1  | 0  |
|    | 11 | 0  | 1  | 1  | 0  |
|    | 10 | 0  | 0  | 0  | 0  |

a=1